Measurement of an Invariant in Directional Couplers

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***Abstract*—It is studied how to measure an invariant of lossless four-port circuits with a reflection symmetry which result in directional couplers. The invariant K determines the isolation port as well as the coupling port with the amount of coupling, and thus the way of coupling is classified into three types corresponding to the cases $K \leq -1$, $-1 < K \leq 1$, and $K > 1$. The value of invariant is easily calculated from S-parameters commonly obtained by a vector network analyzer in the microwave frequency band. It can be used for predicting the transmission characteristics of a directional coupler without building the matching circuits.**

I. Introduction

Among various structures of a directional coupler, K. Araki and Y. Naito have recently found an invariant for a 4-port circuit with one reflection symmetry that includes most of microstripline-type couplers [1]. The invariant is given by the components of the transfer matrices for even and odd mode excitations of the circuit. This value is kept constant independent of external circuits connected afterwards, e.g., matching circuits and uniquely determines the transmission coefficients of the possible directional coupler with perfect matching. We will show a method of measuring this invariant and discuss the property of directional couplers in conjunction with the invariant.

II. The Invariant and Its Measurement

According to Araki and Naito, an invariant is deduced for a lossless 4-port circuit that has one reflection symmetry as shown in Fig.1. We will have odd or even mode excitation, by short- or open-circuiting the symmetry plane, respectively. If each component of the transfer matrix for both excitation is known, the invariant is given by

$$K = (D^o A^e - B^o C^e - C^o B^e + A^o D^e)/2, \qquad (1)$$

where A D are the components of the transfer matrix (F-matrix), and the superscripts o, e designate odd and even mode excitation, respectively.

Their theory claims the following.

1) Any 4-port circuit with one reflection symmetry can be matched at all the ports and have one isolation port, that is, always constitutes a directional coupler.
2) The location of isolation port and amount of coupling are decided by the K value given in (1).

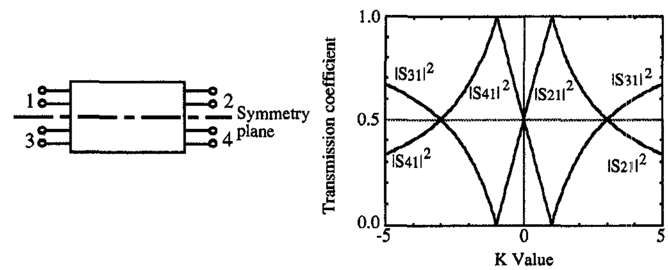

Fig. 1. Intrinsic transmission characteristics of a directional coupler when excited from port 1. Three regions of K-1, -1<K1 and 1<K correspond to three types of coupler having isolation port 2, 3 and 4, respectively.

3) K value does not change if we connect an additional circuit insofar as it does not violate the original circuit symmetry.

The way how the amount of coupling is determined is summarized in their Table I, and, here we will draw it in a figure, instead of a table, for more intuitive understanding (Fig.1). The input signal is assumed to be given in port 1. Directional couplers are seen to be divided into three types according to the K value. When $K > 1$, it represents a "backward coupler," which is common in microstrip-line configuration. The diagonal port (#4 in Fig.1) is isolated in this case. In case that $-1 < K \leq 1$, it will become a "forward coupler" which is frequently encountered in a conventional waveguide coupler or an optical dielectric coupler. The third case where $-1 \geq K$, does not seem to be used in microwave systems, but was studied as a "dc isolation coupler," whose name comes from the fact that the input signal does not go out from the main line. [2]

Considering that the S parameters are the most common circuit parameters measured in the microwave frequency band, it would be more convenient to rewrite the K value in (1) from the transfer matrix coefficients to those of the scattering matrix. Relying on two-port circuit theory, the relations for the circuits shown in Fig. 1 are given

$$\begin{aligned} A^i &= \left(1 - S^i_{11}S^i_{22} + S^i_{12}S^i_{21} + S^i_{11} - S^i_{22}\right)/2S^i_{21} \\ B^i &= \left(1 + S^i_{11}S^i_{22} - S^i_{12}S^i_{21} + S^i_{11} + S^i_{22}\right)/2S^i_{21} \\ C^i &= \left(1 + S^i_{11}S^i_{22} - S^i_{12}S^i_{21} - S^i_{11} - S^i_{22}\right)/2S^i_{21} \\ D^i &= \left(1 - S^i_{11}S^i_{22} + S^i_{12}S^i_{21} - S^i_{11} + S^i_{22}\right)/2S^i_{21}, \qquad (2) \end{aligned}$$

where the superscript i designates odd or even mode excitation. Now we will substitute (2) into (1) and obtain the following relation:

$$K = [(S^e_{11} - S^o_{11})(S^o_{22} - S^e_{22}) + S^o_{12}S^o_{21} + S^e_{12}S^e_{21}]/2S^e_{21}S^o_{21}. \qquad (3)$$

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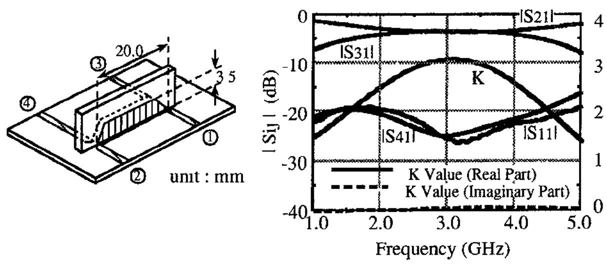

Fig. 2. The structure, transmission characteristics and K value of a VIP directional coupler. The relative permittivity of the vertical and horizontal substrate are 2.6 and 4.2, respectively.

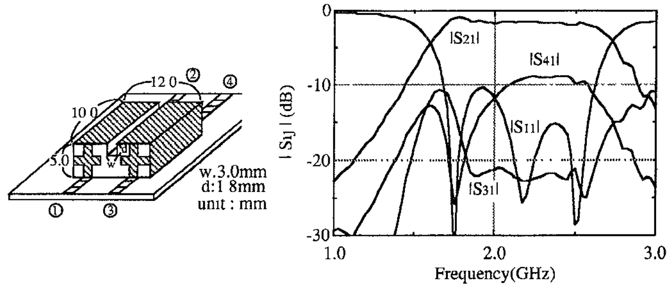

Fig. 3. The structure and transmission characteristics of a dielectric waveguide directional coupler. The relative permittivity is 93.

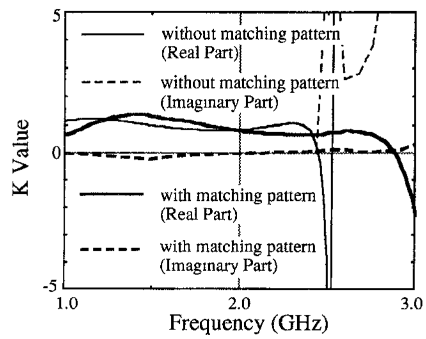

Fig. 4. K value of the dielectric waveguide directional coupler shown in Fig. 3.

The S-matrix coefficients for odd or even mode are related to the original four-port S-matrix coefficient as

$$\begin{aligned} S_{11}^e &= S_{11} + S_{31} & S_{11}^o &= S_{11}^o - S_{31} \\ S_{22}^e &= S_{22} + S_{42} & S_{22}^o &= S_{22} - S_{42} \\ S_{12}^e &= S_{12} + S_{32} & S_{12}^o &= S_{12} - S_{32} \\ S_{21}^e &= S_{21} + S_{41} & S_{21}^o &= S_{21} - S_{41}. \end{aligned} \tag{4}$$

Thus, putting (4) into (3), one will finally have the relation

$$K = \frac{S_{21}S_{12} - 2S_{31}S_{42} + S_{41}S_{32}}{S_{21}^2 - S_{41}^2}. \tag{5}$$

It shows that K value is known only by the output responses when the input signal is given at port 1 and 2. Because of the symmetry, the responses for the excitation of port 3 or 4 are not needed.

It is to be noted that K is a real quantity although S parameters are complex. Considering that the theory is based on the assumption that the circuit is lossless, the diagonal components A and D of the transfer matrix are real, whereas the off-diagonal components B and C are imaginary. Therefore, (1) evidently gives a real value. The expression of (5), however, is not explicitly real, and thus the imaginary part of K calculated by (5) will show the deviation from the losslessness and the symmetry of the fabricated circuit.

III. Experimental Results on Some Typical Examples

Type I ($K > 1$)

The first example is a V.I.P. (vertically installed planar) directional coupler shown in Fig. 2 [3]. This configuration is intended to have a strong coupling, aiming at a 3-dB hybrid. The scalar response in Fig. 3 indicates that it is a wide-band 3-dB hybrid having an isolation port #4 (input port is assumed #1 again). The K value in Fig. 2 corresponds to the characteristics just given, giving the value 3 at the center frequency. The structures as shown in Fig. 2 have one more symmetry plane than is necessary for the theory. It, therefore, gives a simpler expression for K value as

$$K = \frac{S_{21}^2 S_{12} - 2S_{31}^2 + S_{41}^2 S_{32}}{S_{21}^2 - S_{41}^2}. \tag{6}$$

Due to the two-fold symmetry, K is calculated from the S parameters only for the excitation of port 1. The curves in Fig. 2 were obtained by (6).

Type II ($-1 \leq K \leq 1$)

A directional coupler basically made of two dielectric rectangular waveguides is the next example (Fig. 3) [4]. Due to the high relative permittivity of 93, the dimension is drastically reduced as shown in the figure. The transition from a microstrip line is made by an electrode patterned on the endface of the device, including a matching circuit.

The straight vertical electrode in Fig. 3 is the basic excitation scheme and the horizontal pattern like a cross acts as a shunt inductor and capacitor. It is understood an 8-dB coupler is created for 2.1–2.5 GHz from the transmission and reflection characteristic shown in Fig. 3. The corresponding K in Fig. 4 is depicted by thick lines, whereas that for no matching pattern is by thinner lines. The value of about 0.7 between 2.1 and 2.5 GHz indicates the coupling should be 8 dB. The fact that K values are not very different below 2.5 GHz between with or without matching pattern confirms that K value is an invariant which is not affected by the external circuits. It is thus clarified that one can predict the intrinsic characteristics of a directional coupler from its S parameters without matching circuits.

IV. Conclusion

A measuring method of an invariant in directional couplers was first described. It determines the isolation port and also the coupling port along with the amount of coupling. Secondly, we have shown two examples describing the usefulness of the invariant. It would be possible to know the potential properties of a directional coupler by the invariant.

V. Acknowledgement

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